WINVEST: An Asset/Liability Management Case
GAMS Summer School
Soren S. Nielsen, Fall 2000.

1 Introduction

The purpose of this case is to familiarize the students with an important area of applied mathematical modeling, portfolio management. The students act as portfolio managers in the insurance company Winvest. Their responsibility is to fund a future liability stream (annuities) by investing in an asset portfolio (mortgage-backed securities). The future prices and returns of the securities, and the future liabilities are assumed to be functions of interest rates in the future, which of course are unknown at present.

The complete model is developed in stages: First, the basic model is developed for the deterministic case where all future prices etc. are known. Then uncertainty is introduced: The basic model is solved for different sets of data, corresponding to different future interest rate scenarios. Naturally, this raises the question of whether the initial portfolios under different scenarios can be allowed to be different, and the model is refined to avoid this.

The model allows rebalancing, i.e., after the initial portfolio has been held for a while and changes in interest rates have been observed, the portfolio can be brought up-to-date. We ask whether it makes any difference for the initial portfolio (with later rebalancing) if there are transaction costs when buying or selling securities.

The main part of the model can be solved using any LP package, for instance Excel 4.0 or Lindo. However, the last questions (on expected utility maximization) require the students to solve nonlinear models, which can be done in Excel 4.0, but not Lindo. Advanced students can of course use GAMS or any other advanced modeling language.

2 The Story

Bill Rice and his investment group at Winvest has just received the good news: Winvest won the contract of managing the annuities for a large group of employees from the local hospital. Bill immediately understood the importance of this: Managing the account, worth about $100 million, could mean a nice windfall to Winvest. Since Bill's group would be managing the account, and since Winvest employed profit sharing, this made Bill very excited. But he was aware that this was a time to be cautious: If this account was mismanaged, he could be looking for a new job in no time. They needed to act quickly; the $100 million check would arrive in two weeks, and had to be invested immediately. Bill realized he didn't know enough of the dynamics of annuity contracts, so he asked Ann to find out how they worked. Also, it was the policy of Winvest to invest in Mortgage-Backed Securities only, and he also know too little about those. "Peter, why don't you see what you can dig up about MBSs?", he asked. "We'll all meet again tomorrow — we've got to get going on this one!"

3 The Next Day

Ann made a very nice presentation, considering the time pressure. "The annuities we deal with here are called SPDAs, for Single Premium Deferred Annuities", she began. "They work like this: An

investor, in this case the hospital's employee group, pays a premium to an insurance company, that is, us. Then when an employee reaches the age of $59\frac{1}{2}$, we have to pay him or her money every month till they, er, pass away". She looked around, it always embarrassed her to talk about these things.

Bill asked, "Sounds good. Now, how old are these people. Are they in good health? I hope not!" "Well", Ann answered, "They are mostly young. None is over 55. In any event, we won't have to worry about paying out any scheduled payments for several years." "Great", interrupted Bill, "then we can just invest the money into something safe and long-term, and we are home free. Right?"

Ann knew it would get complicated. "Not really. You see, while we have their money we have to pay interest on the account. And if we don't pay enough, then they can lapse. In other words, each policy holder may decide to take his or her portion of the money out and invest it somewhere else. In fact, I spoke with Gary from Analytic Support. He has a lapse model which is based on interest rates and our crediting rates. If interest rates are higher than our crediting rate, then they begin to lapse, and we have to pay them." Bill again: "But then we just pay them a higher interest. What's the problem?" "The problem is", Ann replied, "that we can't pay too much. That's expensive. Besides, we have to invest the money somewhere, and who says that we will have the cash available when or if interest rates rise? After all, short-term investments pay less than long-term ones, don't they"

Bill was beginning to understand. "So, if we want to keep their money, then we have to pay them more if interest rates rise, but the money is tied up. What if we invest the money into something which gains in value when interest rates rise?" He turned to Peter: "What do we know on the asset side, Peter?"

"Well, we have a certain universe of mortgage-backed securities to invest in, both PTs, IOs and POs. Gary also has a prepayment model, and he says..." — "Hold your horses, Peter, I don't understand a word of what you are saying. PTs? IOs? And what are these mortgage things?", Bill objected.

"I better start from the beginning. Do you have a house, Bill?" "Yes". "Where did you get the money from when you bought it?" "From the bank, of course". "But where did the bank get it from?" "I don't know. Probably from people who put their money there, for all I know." "Well, you may be right. But very often, banks and other mortgage lenders won't accept such a large liability as a mortgage from somebody as, you know, as, er, well who could lose his job any time...". Bill frowned, but didn't say anything. "Anyway, what the bank would usually do is to resell your mortgage to Fannie Mae or Ginnie Mae. They buy large numbers of mortgages, pool them together, and then turn around and sell securities which are backed by the mortgages."

Once again, Bill was lost. Ann smiled, she had followed Peter so far. "Who are these characters, the Mae siblings? I've never heard about them", Bill said. At least he knew what a security was: A certificate which you buy, and then you receive money in the future. Much like a bond, except it wasn't given how much money you would receive. In rare cases, the issuer even defaults, so you don't get anything at all. Securities are riskier than bonds, he knew, (this was before "junk bonds") but of course their returns are much higher, on the average.

"Fannie Mae and Ginnie Mae is just the way we pronounce FNMA and GNMA, the Federal National Mortgage Association and the Government, er, I'm not sure what GNMA stands for. But these are giant organizations which were founded with the purpose of lending mortgage money and selling MBSs".

Bill was getting tired. "What are the risks with these securities?" Peter knew the answer to that. "There is virtually no default risk because of mortgage pooling. But there is prepayment risk. Remember, whenever the homeowners pay their mortgage, that money is passed on to the security owners. But if a homeowner refinances his mortgage, or sells his house and moves away, then you

(the security owner) suddenly get a lot of money faster than you thought." "Sounds good to me", said Bill. "Not so fast, Bill. You see, homeowners often refinance when interest rates drop, and that's exactly when you don't want your money back — where are you going to reinvest it when rates are low? Anyway, let me explain some more. You see, there are different kinds of MBSs. The one where all the money which the homeowner pays is passed through to the security owner is called a PT, for Pass Through. So the owner of a PT gets both the principal and the interest parts of the mortgage holders' payments. But you can also buy so-called derivative securities, ones where you get only the interest part, or IOs, and ones where you get only the principal part, or POs, for Interest-Only and Principal-Only, of course. These latter two types of securities are much more volatile than PTs. For instance, if interest rates go down —".

Bill interrupted, "I know! Then homeowners tend to refinance their mortgage at lower rates. The owner of a PO then receives the remaining principal value in the house right away. And the owner of the IO, he, er, what does he get?" "Not much. He doesn't get any of the principal, and since the principal is now paid out, he also doesn't get any interest in the future." "So he should sell the IO", Bill suggested intelligently. "Yes, but he should sell it *before* rates drop — because afterwards it isn't worth much", Peter smiled.

Bill signed. "How do we get anything concrete to work with? This is all getting very confusing. Any ideas anybody?" Ann leaned forward: "Yes, Gary also has a prepayment model. He can prepare data for us. We tell him which securities we are interested in, and under what future interest rate scenarios. Then he runs his model and tells us the future cash-flows and prices of the securities. He can run his lapse model at the same time, so we can get all the data we need."

"OK, let's call it a day. My head is spinning, and I need to sleep on this. Let's meet same time tomorrow." Bill was concerned. This was more difficult than he had thought. He hoped Gary knew what he was doing.

Facts:

1. The contract (and the check which Bill's group will receive) is for $100,000,000.
2. The portfolio will be restricted to containing PTs, IOs and POs, or be held in cash.
3. They would rely on Gary's prepayment and lapse models.
4. They would have to come up with interest rate scenarios for Gary.

4 The Second Day

They met early. Ann started the meeting. "So, it seems that we need to come up with some securities that we're interested in, and some future interest rate scenarios. Bill, what's the time frame involved?" Bill: "Good question. We receive the check next week, and have to decide on an investment by then. Then we can't do anything for 6 months — we are not allowed to touch the portfolio. You know, they try to keep broker costs down. Anyway, 6 months from now we can rebalance the portfolio if we need to. Also, on that same day we'll know if anybody at the hospital decided to lapse, and how much we'll have to pay them. After we pay them and reoptimize the portfolio, we can't touch it for another 6 months. And that's as long as we can plan ahead, as far as I can see: 12 months." Bill liked his own use of the word "reoptimize". But the others weren't impressed; by now it was obvious that some kind of optimization would be necessary.

After some discussion, the group decided on 4 interest rate scenarios. The scenarios were based on "worst cases", because they felt that worst cases would somehow lead to a portfolio which was

Security	WAC	WAM	OAP	Price
FNMA-8.00-PT	8.750	330	1.067180	94.8438
FNMA-10.00-PT	10.750	354	1.102980	104.0625
GNMA-12.00-PT	12.500	292	0.960422	111.7812
FNSTR-1-PO	9.69	298	0.815915	58.0940
FNSTR-1-IO	9.69	298	1.467120	42.2380
FNSTR-2-PO	10.58	294	0.776155	64.7190
FNSTR-2-IO	10.58	294	1.584240	39.7390
FNSTR-7-PO	9.60	120	0.886932	55.5000
FNSTR-7-IO	9.60	120	1.457880	42.3770
FNSTR-70-PO	10.50	342	0.957392	59.4060
FNSTR-70-IO	10.50	342	1.165050	45.8330
FNSTR-90-IO	9.71	345	1.286940	45.0400

Table 1: Mortgage-backed securities in the portfolio universe. WAC: weighted average coupon rate. WAM: weighted average maturity (months). OAP: Option adjusted premium. From April 26, 1991.

well "hedged". They felt that they didn't quite understand what that meant yet, but they had to give Gary something to work with. The four scenarios were dubbed uu, ud, dd, du: For instance, uu was the scenario where interest rates go up as fast as possible during the first 6 months, and again during the second 6-month time period, ud was the scenario where rates go up during the first period, then down again as fast as possible, and so on. They knew that "up as fast as possible" and "down as fast as possible" were well-defined terms under some model of the "term structure of interest rates", but decided to leave the details of that to Gary.

Facts:

1. There are three time points where things happen: Next week, when the initial portfolio has to be purchased, 6 months later when the first liability due to lapse is known, and the portfolio can be rebalanced, and 6 months after that again, when the total asset/liability situation is evaluated.
2. 4 interest rate scenarios will be used: uu, ud, dd, du.
3. 4 MBSs would be used for possible investment, besides investment in cash: Two IOs, called FNSTR-2-IO and FNSTR-90-IO, and two POs: FNSTR-7-PO and FNSTR-70-PO. The data for some securities are given in Table 1.
4. The group's performance would be evaluated based on the asset/liability situation 12 months from now. More precisely, the value of the assets, as if they were sold, and of the liabilities, as if *they* were sold (to another insurance company), would be calculated, and the group's future based on the difference.

5 The Third Day

The data from Gary had arrived. He had even simplified things a little, after having discussed with Ann exactly what they needed. Running his prepayment and lapse models hadn't been complicated.

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----   4822 PARAMETER YIELD

                 UU          UD          DD          DU

IO2 .T0     1.104439    1.104439    0.959238    0.959238
IO2 .T1     1.110009    0.975907    0.935106    1.167817
PO7 .T0     0.938159    0.938159    1.166825    1.166825
PO7 .T1     0.933668    1.154590    1.156536    0.903233
PO70.T0     0.924840    0.924840    1.167546    1.167546
PO70.T1     0.891527    1.200802    1.141917    0.907837
IO90.T0     1.107461    1.107461    0.908728    0.908728
IO90.T1     1.105168    0.925925    0.877669    1.187143

CASH.T0     1.030414    1.030414    1.012735    1.012735
CASH.T1     1.032623    1.014298    1.009788    1.030481


----   4822 PARAMETER LIAB

            UU          UD          DD          DU

T1   26.474340   26.474340   10.953843   10.953843
T2   31.264791   26.044541   10.757200   13.608207

VAL  47.284751   49.094838   86.111238   83.290085
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Table 2: Output from Gary's Prepayment and Lapse Models

Ann brought the output directly to the group's third meeting.

"Here's the output from Gary's prepayment model. It shows, for each of our four securities, the yield during the first 6-month period, and during the second 6-month period, for each of our four interest rate scenarios. It's quite a lot of data, but it's really simple to understand once you get used to it." Bill looked at it. "But you said that we needed prices — how do we know how much the securities cost, and what they are worth after 6 and 12 months? And we also need the cash-flows, I mean, how much do the homeowners pay? Can we do anything without those data?"

Ann explained that Gary had actually helped them by combining some figures into the "Yields" shown in the table. "You see, what Gary calls "yields" is actually the change in value of a holding of each particular security under each scenario. For instance, if you invest $1 in FNSTR-2-IOs to begin with, then if interest rates go up (the uu or ud scenarios), that investment will be worth $1.104439 after 6 months. But if interest rates go down (the dd or du scenarios), it'll be worth only $0.959238. And so on. All yields that are labeled "T0" have to do with the first 6-month period, and the data labeled "T1" have to do with the second 6-month period. I think this is really all we need."

"I see that the liabilities are there, too", Bill said. "Yes, and they are just as easy to understand. Here, "T1" means that we have to pay $26.474340 back to the hospital's employee group for each $100 SPDA they jput into their SPDA initially, 6 months from now if interest rates go up, and so on." "What's the VAL entry?" "Those figures are the values of the remaining outstanding liability

after 12 months. I don't quite understand how Gary's model finds these, some sort of expected net present value a year from now. But he says that they are fair estimates of the future values we need in order to evaluate our net position."

Facts:

1. The group now knows, for each interest rate scenario, for each time period, and for each security, how much the value of an investment in that instrument will change during that time period under that scenario and time period.

2. The group also knows, for each scenario and each time period, what the lapse liability will be, as well as the expected value of the remaining outstanding liability after 12 months.

3. The four interest rate scenarios are equally likely.

6 Putting the Model Together

It was time to get down to the dirty details. Bill was still a bit concerned about the seeming complexity of the task ahead of them. He suggested several simplifications to the initial model, and the group decided that his suggestions made sense. They usually decided that; after all, Bill was the boss.

First, it was decided to ignore the liabilities initially. They seemed like unnecessary complications, but at the same time, the group felt that once they understood the model without liabilities, they could probably easily add them in. Second, they decided that they would simply find the optimal portfolios under each of the four scenarios. They could pretty much do that by inspection. In a few minutes, they had even calculated the expected return of their investment. But they knew that their result didn't make much sense. For one thing, they had ignored the fact that buying and selling securities incurs transaction costs. Besides, the four first-stage portfolios which were optimal for each of the four scenarios weren't all the same, so they couldn't actually go ahead and *do* this. But at least it was a beginning.

The First Model:

1. Find the portfolio which is optimal for the first and second stage decisions under each of the four scenarios and calculate the expected return. Ignore for the time being the liabilities and transaction costs.

2. Define a model which would solve this problem for each scenario separately. Specify any assumptions you make. Define the decision variables carefully. Do you think the solution would change if transaction costs were included? Liabilities?

3. Implement this model using your favorite software. Keep in mind that the implementation should be "extendable". You may later have to solve the corresponding model for all four scenarios, or in other variants.

4. Solve the model with transaction costs. There is a transaction cost (loss) of 1% whenever a security is sold (not when purchased). How does this change the solution?

7 Refining the model

It was now time to solve a more realistic version of the model. The group was happy with the progress they had made the last few days. They had deciphered Gary's data, and they had solved a version of the model, a simple-minded version perhaps, but still. Furthermore, they had a computer running their model. Ann had the precious floppy, where they had everything stored (after having left the computer for a short break and found everything erased when they came back, they now saved their stuff every fifteen minutes or so). Ann had even solved this simple model with transaction costs incorporated — she said that had been easy, but she would always say things like that.

The main problem with the model now was that is found a solution for each scenario individually. Without knowing the future, that wasn't very useful. Clearly, somehow they had to link the four scenario solutions together to obtain a unique first-period decision.

The Second Model:

1. Change the first model so that the same first-period decision is reached for all scenarios. In other words, the model should return an initial, first-period portfolio which does not depend on the individual scenario, but which does result in the maximal expected return when allowing for (scenario-dependent) rebalancing at 6 months. Comment on the new solution and the final wealth position compared to the previous model. For instance, is the new solution more "risky"? How can one quantify the risk?

2. Compute the Expected Value of Perfect Information about the first-period interest rates.

3. Now incorporate the stochastic liabilities into the model. Again, comment on the new solution.

8 Reducing the Risk

Bill was satisfied with the progress of his group, but not entirely with the results of the model. Looking at Ann's output, the expected returns of their investment looked good compared to the final value of the liabilities, but it seemed to him that if interest rates were to go up and up, he'd be in serious trouble. That's where the insured hospital employees were most likely to lapse, he knew, but he was surprised that the impact was so big. There had to be a better way to hedge against this.

Peter had an idea. "Why don't we use a utility model instead of solving for the maximum expected return? Maybe if we try to maximize the expected *utility* of the final position, we'll get a better result." "What do you mean, utility?", asked Bill. He had been thinking about using a mean-variance model, but couldn't make sense of that for the dynamic model they were using.

Peter explained. "Utility is a way to measure a distribution of returns, or, if you will, is a weighted average where bad returns receive a higher weight. The expected return model is, strictly speaking, a utility model, but with all returns weighted equally. What you do is define a utility function, and then have the model maximize the expected utility of final wealth."

Bill and Ann looked at each other. They both remembered having heart about this somewhere before, but they had never quite understood it. Maybe this was their chance!

The Third Model:

1. What do you think of Bill's idea of using a mean-variance model?

2. Solve the previously developed model (with transaction costs and liabilities incorporated) but with the additional constraint that the final value of assets must be at least as large as the value of the final liabilities (VAL) under each scenario. Comment on the solution.

3. Develop and solve a utility maximizing model. The utility function should correspond to a risk-averse attitude, i.e., be concave. The log function (the natural logarithm) is popular in the finance world. This function assigns very large, *negative* utilites to returns near zero, and in particular prohibits non-positive returns. **Hint 1.** If you use the log utility function, be sure that the expression of which you take the log is positive. This can sometimes be done by giving some variables positive lower bounds. **Hint 2.** Let w_s be the *final wealth*, i.e., the final value of assets minus final value of liabilities, under scenario s, where s is uu, ud, dd or du. An expression for expected utility of final wealth is then

$$\frac{1}{4}\sum_{s\in\mathcal{S}}\log w_s.$$

Notice that an equivalent expression is

$$\frac{1}{4}\log\left(\prod_{s\in\mathcal{S}} w_s\right).$$

Instead of maximizing this function, you will get the same optimal solution (although a different objective value) by just maximizing

$$\prod_{s\in\mathcal{S}} w_s.$$

(The symbol Π means "product", just like Σ means "sum"). This objective results in the same solution because log is, as all utility functions, increasing. Note that the variables w_s have to be strictly positive for this to work.

4. In reporting results of utility models, it's usually a bad idea to report the utility value obtained, since it doesn't mean much by itself. Instead, calculate and report the *certainty-equivalent return,* i.e., the fixed return which has the same utility as the model's result. Also, for the result of any model you can still calculate the expected (risk-neutral) return. In other words, make sure that the way you report the results makes sense to non-technical readers.

9 Conclusion

It was a dark and stormful night. Bill, Ann, Peter and Gary didn't care. They enjoyed their visit to Peter's apartment. He was actually an excellent cook. Nothing fancy: T-bone Steak, Sauce Bernaice, baked potatoes with parsley and garlic butter, Beaujolais, the works. The group felt pretty good about themselves as they discussed the last couple of weeks' work. It had been hard, but they felt happy and secure with their investment, and also thought they had learned something. Now, all they had to do was watch the interest rates!

Final Comments:

1. You are invited to comment on this case: Its appropriateness to the course, difficulty and anything else you might like.

GAMS Summer School 2002:
Questions for the Winvest Case

Søren Nielsen

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The background information for the case is given in the note (winvest.pdf) on the CD. However, instead of answering the questions given in that note, do the following ones. These questions will guide you through the modeling process in an easy way.

1. *The Very First Model:* Model the UU scenario as a small network model (see the figure), maximizing final wealth. Do not use transaction costs or liabilities. The result should be trivial to verify.

2. *The 4-Scenario Model:* Using a scenario index "s" on all constraints and variables, extend your single-scenario model to contain all 4 scenarios.

   The objective should be to maximize the expected (average) final wealth. Again, the results are trivial to verify.

3. *Transaction Costs:* Incorporate transaction costs (1% on *sales*). This changes the solution under some (all?) of the scenarios. How? This is almost trivial to verify. Now, you should have confidence in your model!

4. *True Stochastic Model:* The initial investments so far were dependent on the specific scenario, which is no good since we don't know which scenario will happen. Find a way to make the initial investments the same no matter what the scenario is. (Hint: You need to make the first-stage decisions equal across scenarios).

5. *Reducing the Risk 1:* A very risk-averse investor would be concerned about the worst outcome, i.e., would like a portfolio that has the *best worst-case outcome*. Design such a model. Compare the solution to the base case (max expected final wealth).

   On the other hand, a very optimistic investor would expect the best case to actually happen, i.e., might *maximize the best-case outcome*. Do not implement

this model, but explain how, or whether, it can be done (as an LP-model). What is the optimal solution?

6. *Reducing the Risk 2:* Finally, solve a utility model by *maximizing expected utility of final wealth*, where the utility function is the logarithm.

   To avoid taking the log of non-positive numbers, put a lower bound of 0.0001 on the final wealth variables.

Note: Stochastic Liabilities and Expected Value of Perfect Information: Don't use these in this model.