of expecting losses is computed as the area below the pdf within the interval $(-\infty, 0)$. In this case, it can be observed in the cdf that this probability is equal to 0.08.

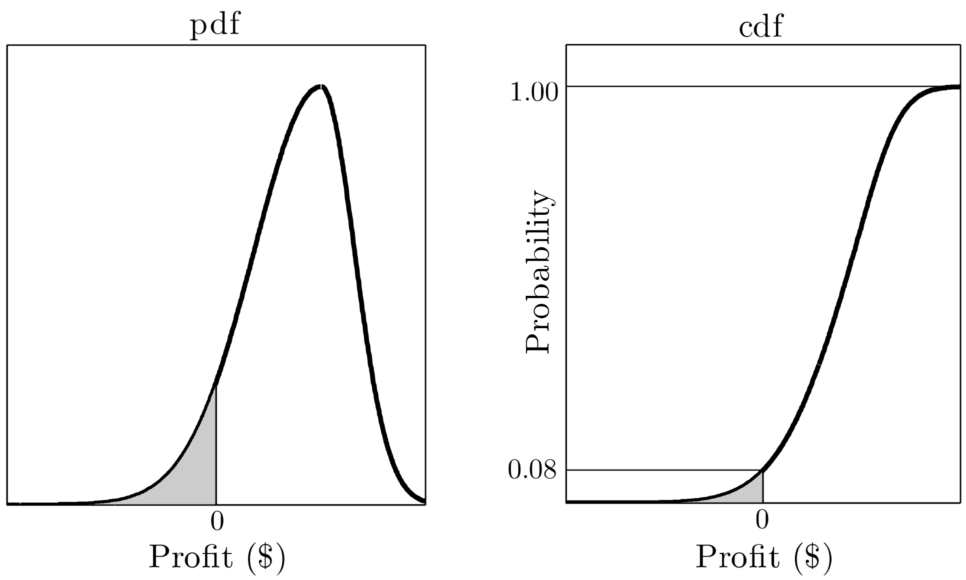


**Fig. 8.10** Illustrative Example 8.8: profit distribution

□

It should be clear that the decisions made by the retailer have a significant influence on the resulting volatility of the profit. For instance, if the retailer only purchases electricity in the pool (risky source) the volatility of the profit is much higher than that if the retailer relies more on the futures market (unrisky source).

In order to consider the volatility of the profit in the decisions made by the retailer, it is appropriate to include a risk measure in the formulation of the problem. Including a risk measure limits the risk of experiencing a profit significantly smaller than the expected one. A well-known risk measure is the Conditional Value-at-Risk (CVaR) [124, 125]. For an $\alpha$ confidence level, the CVaR is approximately the expected profit of the $(1-\alpha) \times 100$ scenarios with the lowest profits. The CVaR is explained in detail in Subsection 4.3.5 of Chapter 4.

Using expression (8.13) for the profit, the CVaR for a confidence level $\alpha$ is computed as

$$\text{CVaR} = \text{Maximize}_{\zeta, \eta_\omega} \qquad \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \, \eta_\omega \tag{8.15}$$

subject to

$$\zeta - \sum_{t=1}^{N_\mathrm{T}} \left( \sum_{e=1}^{N_\mathrm{E}} R^\mathrm{R}_{et\omega} - C^\mathrm{P}_{t\omega} - C^\mathrm{F}_{t} \right) \leq \eta_\omega, \ \forall \omega \tag{8.16}$$

$$\eta_\omega \geq 0, \ \forall \omega. \tag{8.17}$$

The optimal value of $\zeta$ represents the greatest value of the profit such that the probability of experiencing a profit less than or equal to $\zeta$ is less than or equal to $1-\alpha$. The auxiliary variable $\eta_\omega$ is equal to the excess of $\zeta$ over the profit in scenario $\omega$, if this excess is positive.

As explained in Subsection 4.3.5 of Chapter 4, the CVaR can be equivalently included in the formulation of the problem as an additional constraint or in the objective function either alone or together with the expected profit.

## 8.6 Formulation

The formulation of the retailer problem is provided below

$$\begin{aligned} &\mathrm{Maximize}_{P^\mathrm{F}_{fj}, \lambda^\mathrm{R}_{ei}, v_{ei}, E^\mathrm{P}_{t\omega}, \zeta, \eta_\omega} \\ &\sum_{\omega=1}^{N_\Omega} \pi_\omega \sum_{t=1}^{N_\mathrm{T}} \Big( \sum_{e=1}^{N_\mathrm{E}} \sum_{i=1}^{N_\mathrm{I}} \lambda^\mathrm{R}_{ei} \bar{E}^\mathrm{R}_{eti\omega} - \lambda^\mathrm{P}_{t\omega} E^\mathrm{P}_{t\omega} - \sum_{f \in F_t} \sum_{j=1}^{N_\mathrm{J}} \lambda^\mathrm{F}_{fj} P^\mathrm{F}_{fj} d_t \Big) \\ &\qquad + \beta \left( \zeta - \frac{1}{1-\alpha} \sum_{\omega=1}^{N_\Omega} \pi_\omega \eta_\omega \right) \end{aligned} \tag{8.18}$$

subject to

$$0 \leq P^\mathrm{F}_{fj} \leq \bar{P}^\mathrm{F}_{fj}, \ \forall f, \forall j \tag{8.19}$$

$$\bar{\lambda}^\mathrm{R}_{ei-1} v_{ei} \leq \lambda^\mathrm{R}_{ei} \leq \bar{\lambda}^\mathrm{R}_{ei} v_{ei}, \ \forall e, \forall i \tag{8.20}$$

$$\sum_{i=1}^{N_\mathrm{I}} v_{ei} = 1, \ \forall e \tag{8.21}$$

$$\sum_{e=1}^{N_\mathrm{E}} \sum_{i=1}^{N_\mathrm{I}} \bar{E}^\mathrm{R}_{eti\omega} v_{ei} = E^\mathrm{P}_{t\omega} + \sum_{f \in F_t} P^\mathrm{F}_{f} d_t + E^\mathrm{PC}_{t}, \ \forall t, \forall \omega, \tag{8.22}$$

$$\zeta - \sum_{t=1}^{N_\mathrm{T}} \left( \sum_{e=1}^{N_\mathrm{E}} \sum_{i=1}^{N_\mathrm{I}} \lambda^\mathrm{R}_{ei} \bar{E}^\mathrm{R}_{eti\omega} - \lambda^\mathrm{P}_{t\omega} E^\mathrm{P}_{t\omega} - \sum_{f \in F_t} \sum_{j=1}^{N_\mathrm{J}} \lambda^\mathrm{F}_{fj} P^\mathrm{F}_{fj} d_t \right) \leq \eta_\omega, \ \forall \omega \tag{8.23}$$

$$v_{ei} \in \{0,1\}, \ \forall e, \forall i \tag{8.24}$$

$$\eta_\omega \geq 0, \ \forall \omega. \tag{8.25}$$

The objective function (8.18) comprises two terms: i) the expected profit and ii) the CVaR multiplied by the weighting factor $\beta$. The factor $\beta$ models the tradeoff between expected profit and CVaR.

Constraints (8.19) bound the power contracted from each block of the forward contracting curve of each contract. Constraints (8.20)-(8.21) identify the block of the price-quota curve associated with each selling price. Constraints (8.22) impose the electric energy balance in each period and scenario. Constraints (8.23) are used to compute the CVaR. Finally, (8.24) and (8.25) constitute variable declarations.

## 8.7 Retailer Example

We present an example of reduced size to illustrate the formulation of the retailer problem described above.

We consider a planning horizon of two hourly periods. Two forward contracts spanning both periods are available. The forward contracting curves corresponding to each contract have two blocks. The parameters defining each contract are provided in Table 8.3.

**Table 8.3** Retailer example: forward contracting curve data

| Contract # | Usage period # | $\lambda^{\mathrm{F}}_{f1}$ (\$/MWh) | $\lambda^{\mathrm{F}}_{f2}$ (\$/MWh) | $\bar{P}^{\mathrm{F}}_{f1}$ (MW) | $\bar{P}^{\mathrm{F}}_{f2}$ (MW) |
|---|---|---|---|---|---|
| 1 | 1-2 | 66.00 | 69.30 | 20 | 20 |
| 2 | 1-2 | 67.00 | 70.35 | 20 | 20 |

The client electricity demand and pool prices are modeled using a set of four equiprobable scenarios, which are shown in Table 8.4. In this example, a single group of clients is considered.

The response of the clients to the price offered by the retailer is represented through the price-quota curve shown in Fig. 8.11. A single price-quota curve is used for all periods and scenarios. As can be seen in Fig. 8.11, if the selling price is smaller than \$65/MWh, the retailer supplies 100% of the client demand. However, if this price is in between 65 and \$75/MWh, the retailer supplies 35% of the total demand of the clients. Likewise, if the selling price is in between 75 and \$85/MWh, the retailer just supplies 15% of the demand, and if the price is above \$85/MWh, the retailer supplies no demand.

Problem (8.18)-(8.25) is set up with a confidence level $\alpha$ equal to 0.95. The resulting problem, characterized by 23 constraints, 24 real variables, and 3 binary variables, is solved using CPLEX 10.2 [142] under GAMS [141].

**Table 8.4** Retailer example: client demand and pool price scenarios

| Scenario # | Client demand (MWh) | | Pool price ($/MWh) | |
|---|---|---|---|---|
| | Period # | | Period # | |
| | 1 | 2 | 1 | 2 |
| 1 | 350 | 325 | 60 | 52 |
| 2 | 365 | 335 | 65 | 55 |
| 3 | 375 | 345 | 74 | 68 |
| 4 | 360 | 340 | 70 | 61 |

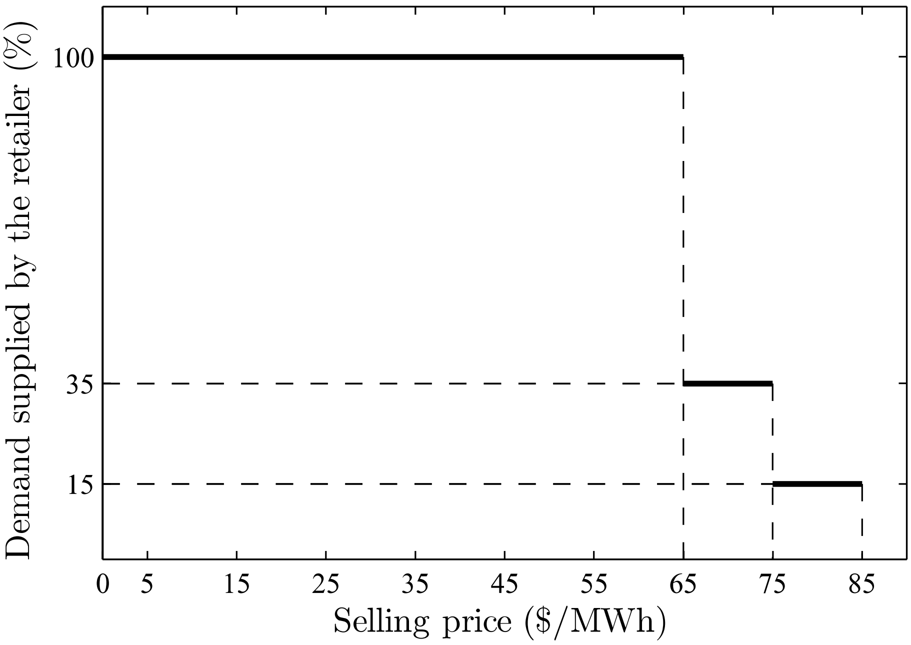


**Fig. 8.11** Retailer example: price-quota curve

The expected profit, profit standard deviation, and CVaR for different values of $\beta$ are presented in Table 8.5. As expected, the solution achieved for $\beta = 0$ attains the highest expected profit and the highest risk, measured as both the standard deviation and the CVaR of the profit distribution. For $\beta = 100$ the expected profit decreases 36.9% to attain a reduction of 98.1% in the profit standard deviation and an increase of 92.9% in the CVaR.

The efficient frontier, representing the expected profit versus the profit standard deviation and the CVaR for different values of $\beta$, is shown in Fig.